MIDTERM 2
MATH 2030 ODE, FALL 2018

Instructions: Solve as many problems or subproblems as you can in the given time. You should show all your work in order to get full or partial credit. You may write on the back of pages and staple additional pages if you run out of space. I recommend first working on whichever problems you know how to solve before spending time on the trickier ones. The raw scores will be curved and you do not necessarily need to solve all the problems to get a good grade. Good luck!!

Problem 1. (6 pts / 100) Consider the ODE

$$y'''(t) + sin(t)y'(t) + 17y(t) = 0.$$

Suppose $f(t), g(t), h(t)$ are solutions, defined for all $t \in \mathbb{R}$. Suppose that we have

$$f(\pi) = 1, \quad f'(\pi) = 2, \quad f''(\pi) = 3,$$
$$g(\pi) = 4, \quad g'(\pi) = 5, \quad g''(\pi) = 6,$$
$$h(\pi) = C, \quad h'(\pi) = 8, \quad h''(\pi) = 9,$$

where C is a real constant. Do $f(t), g(t), h(t)$ together form a fundamental set of solutions? *Note: your answer may depend on C.*

Problem 2. (15 pts / 100) Find the general solution to the ODE

$$t^2 y''(t) + 2t y'(t) + y(t) = 0, \qquad t > 0.$$

Note: your solution should be valid for all $t > 0$.

Problem 3. (20 pts / 100) Find the general solution to the ODE

$$y^{(6)}(t) - 6y^{(5)}(t) + 11y^{(4)}(t) - 6y'''(t) = 0.$$

Problem 4. (20 pts / 100) Find a particular solution to the ODE

$$y^{(5)}(t) - y'(t) = e^t \sin(t) + 3\cos(t) + t^2 - 1 + e^{-t} + e^{7t}.$$

You may leave your answer in terms of a finite number of undetermined constants, for example $y(t) = A\sin(t) + Be^t$.

Problem 5. (20 pts / 100) Consider the ODE

$$y''(t) + ty'(t) + t^2y(t) = 0.$$

Given that there is a power series solution of the form $y(t) = \sum_{k=0}^{\infty} a_k t^k$ with $a_0 = a_1 = 3$, find a_2, a_3, a_4, a_5.

Problem 6. (7 pts / 100) Consider the ODE

$$t(t+1)^3(t+2)^2y''(t) + t(t+1)(t+2)y'(t) + ty(t) = 0.$$

Find the singular points, and classify them as regular or irregular. *Note: you do not need to justify your answer, although it could help you get partial credit.*

Problem 7. (12 pts / 100) Consider the ODE

$$2t^2y''(t) - 3ty'(t) + 2y(t) + 2ty(t) = 0, \qquad t > 0.$$

Given that there are two linearly independent solutions $y_1(t), y_2(t)$ of the form $y_1(t) = t^{r_1}(1 + \sum_{k=1}^{\infty} a_k t^k)$ and $y_2(t) = t^{r_2}(1 + \sum_{k=1}^{\infty} b_k t^k)$, find r_1 and r_2. *Note: this question is asking you to find the exponents of the singularity at $t = 0$. You do not need to find the coefficients a_k, b_k.*