MIDTERM 1
MATH 2030 ODE, SPRING 2018

Instructions: Solve as many problems or subproblems as you can in the given time. You should show all your work in order to get full or partial credit. You may write on the back of pages and staple additional pages if you run out of space. I recommend first working on whichever problems you know how to solve before spending time on the trickier ones. The raw scores will be curved and you do not necessarily need to solve all the problems to get a good grade. Good luck!!

Problem 1. (20 pts / 100) Consider the following ODE for a function $y(t)$:

$$2t - y(t) + 2y(t)y'(t) - ty'(t) = 0.$$

Find the general solution for $y(t)$. Your answer should involve one arbitrary constant and may be left in implicit form.

Problem 2. (1) (10 pts / 100) Find the general solution to the ODE

$$y'(t) + y(t) = e^t.$$

Your answer should involve one arbitrary constant.

(2) (10 pts / 100) Find the solution to the initial value problem

$$\begin{cases} y'(t) + e^t y(t) = 1 \\ y(t_0) = y_0. \end{cases}$$

You may leave your answer in the form of a definite integral (e.g. something like $\int_{s=t_0}^{s=t} s^{17}\sin(s)ds$). *Hint: you may encounter the function* $\exp(e^t)$, *which is perfectly fine.*

Problem 3. (12 pts / 100) Consider the initial value problem

$$\begin{cases} y'(t) = (1 + y(t))^2 \\ y(0) = y_0. \end{cases}$$

Find the solution, and state the maximal interval for t on which the solution is defined. (For example, $(-\infty, \infty)$ would mean the solution is defined for all t). Your answer may depend on y_0. *Hint: does $y(t)$ have a vertical asymptote? Does it occur for $t < 0$ or $t > 0$? What happens if $y_0 = -1$?*

Problem 4. (1) (14 pts / 100) Find the general solution to the ODE

$$y''(t) + y'(t) + y(t) = 0.$$

Your answer should be a real-valued function and should involve two arbitrary constants.

(2) (14 pts / 100) Find the general solution to the ODE

$$y''(t) - 2y'(t) + y(t) = 0.$$

Your answer should be a real-valued function and should involve two arbitrary constants.

Problem 5. (1) (6 pts / 100) Consider the autonomous ODE given by

$$y'(t) = -y^3 + 7y^2 - 10y = 0.$$

Find the equilibrium points and classify them as stable, unstable, or semistable.

(2) (8 pts / 100) Consider the same ODE as in part (a), now with the initial condition $y(0) = y_0$. For which y_0 (if any) does there exist some $t \in (0, \infty)$ such that $y(t) = \frac{1}{2}y_0$?

(3) (6 pts / 100) Give an example of an autonomous ODE of the form $y'(t) = F(y)$ which has unstable equilibria at $y = 1$ and $y = 2$ and a semistable equilibrium at $y = 3$. *Hint: your ODE may also have additional equilibria.*