PROBLEM SET #1

Problem 1. *How many partitions are there of the set* $\{1, 2, 3, 4, 5, 6\}$*?*

Solution 1. *The number of partitions of a set with* n *elements (*$n = 6$ *in this case) is called a* Bell number*. These numbers show up all the time in different parts of mathematics (see for example Wikipedia as a starting point). To count, let's first count the number of different types of paritions. For example, if a partition has one part of size* 3 *and three parts of size* 1*, we'll write* $3 + 1 + 1 + 1$*. The possibilities are:*

(1) 6
(2) 5+1
(3) 4+2
(4) 3+3
(5) 4+1+1
(6) 3+2+1
(7) 2+2+2
(8) 3+1+1+1
(9) 2+2+1+1
(10) 2+1+1+1+1
(11) 1+1+1+1+1+1.

Now, for example, how many partitions of type $2 + 2 + 1 + 1$ *are there? For have:* $\{1, 2\} \sqcup \{3, 4\} \sqcup \{5\} \sqcup \{6\}$ *and* $\{1, 3\} \sqcup \{2, 5\} \sqcup \{4\} \sqcup \sqcup \{6\}$ *and so on.*