PROBLEM SET #1

Problem 1. *Let A and B be sets. Prove the following*

(1) *a map $f : A \to B$ is injective if and only if it has a left inverse*
(2) *a map $f : A \to B$ is surjective if and only if it has a right inverse*
(3) *if A and B of finite sets of the same cardinality, a map $f : A \to B$ is bijective if and only if it is injective.*

Problem 2. *Given an example of*

(1) *a map $f : \mathbb{Z} \to \mathbb{Z}$ which is injective but not surjective*
(2) *a map $f : \mathbb{Z} \to \mathbb{Z}$ which is surjective but not injective*
(3) *a bijection $f : \mathbb{Z} \to \mathbb{Z}$ which is not the identity map.*

Problem 3. *Let A be a set, and let $\{A_i : i \in I\}$ be a partition of A (here I is some indexing set). Prove that there is an equivalence relation $\sim$ on A whose equivalence classes are precisely the sets A_i for $i \in I$. Note: for this exercise you should be explicit about checking the three conditions for $\sim$ to be an equivalence relation.*

Problem 4. *How many partitions are there of the set $\{1, 2, 3, 4, 5, 6\}$?*

Problem 5. *Let $\mathbb{Z}$ denote the set of integers and let $\mathbb{Q}$ denote the set of rational numbers. Does there exist a bijection $f : \mathbb{Z} \to \mathbb{Q}$? You should explicitly justify your answer.*

Problem 6. *Write out the complete multiplication table for S_3, the symmetric group on three elements. Note: it should be a 6×6 table. You may name the elements of S_3 however you'd like, as long as your naming scheme is clearly explained.*

Problem 7. *Write out the multiplication table for a group of order three. Explicitly verify that the three axioms of a group are satisfied.*

Problem 8. *Prove that every group of order three is abelian.*

Problem 9. *Write out the multiplication table for a group of order five, with the elements are written as a, b, c, d, e. Is your group abelian? (You do not need to prove that the group axioms are satisfied.)*

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