

MIDTERM 1 TOPICS

Here is a list of topics covered by the first midterm. Note that this list is not necessarily complete, it is just meant to give you a rough sense of the main topics we've covered so far.

- basics of group theory: definition of a group, group multiplication tables, checking that a candidate is a group, etc
- subgroups, cosets, Lagrange's theorem, order of elements in a group
- cyclic groups and their properties, the additive group $\mathbb{Z}/(n\mathbb{Z})$ of integers modulo n , the multiplicative group C_n of n th roots of unity
- the multiplicative group $(\mathbb{Z}/n\mathbb{Z})^\times$ of invertible integers modulo n , Euler's totient function, Fermat's Little Theorem
- symmetric groups, transpositions, cycle decompositions, sign of a permutation, the alternating group
- homomorphisms, isomorphisms (e.g. determining whether two groups are isomorphic)
- dihedral groups D_{2n} , matrix groups (e.g. $GL(3, \mathbb{R}), SO(3)$)
- normal subgroups, quotient groups
- direct products of groups
- kernel and image of a homomorphism
- normalizers, centralizers, center of a group, subset HK determined by subgroups H and K
- first, second, and third isomorphism theorems¹

Regarding the format of the exam, you should be prepared to see various types of questions: true / false, multiple choice, short answers, and short proofs. You will need to work fairly quickly during the exam and as a rule of thumb you should first answer the questions you find easier before moving on to the more challenging ones. A few tips for preparing:

- have a good "toolkit" of groups and subgroups so that you can produce various interesting examples and counterexamples
- try to understand the groups we've seen as best you can, i.e. what are the orders of elements, what are their subgroups, which of those are normal, etc
- be able to give proofs of various lemmas and propositions we've encountered in class, and also similar ones appearing in the textbooks and their exercises.

Date: October 4, 2019.

¹Note: I will not directly test you on the fourth isomorphism theorem on this midterm, although it may certainly appear on the second midterm or final exam.