635 FALL 2024 PROBLEM SET #2

Problem 1.

(a) Write $\frac{13}{5}$ as a continued fraction $a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\cdots + \cfrac{1}{a_n}}}}$.

(b) Write $\frac{13}{5}$ as a negative (a.k.a. Hirzebruch–Jung) continued fraction $b_0 - \cfrac{1}{b_1 - \cfrac{1}{b_2 - \cfrac{1}{\cdots - \cfrac{1}{b_n}}}}$.

Problem 2. Let σ be a rational polyhedral cone in $N_{\mathbb{R}}$, with associated affine toric variety V_σ.

(a) Show that if σ spans $N_{\mathbb{R}}$, then the action of the torus T_N on V_σ has a unique fixed point, namely the distinguished point x_σ.

(b) Show that if σ does not span $N_{\mathbb{R}}$, then there is no fixed point in V_σ.

Problem 3. Let σ be a rational polyhedral cone in $N_{\mathbb{R}}$, let τ be a face of σ, and suppose that $v \in N$ lies in the relative interior of τ. Let $\lambda_v : \mathbb{C}^* \to T_N$ be the cocharacter associated to $v \in N$. Show that $\lim_{t\to 0} \lambda_v(t)$ equals the distinguished point $x_\tau \in V_\sigma$. *Hint: if you get stuck you may wish to consult Fulton §2.3.*