535A SPRING 2023 PROBLEM SET #6

Problem 1. Lee *second edition* 6-1

Problem 2. Lee *second edition* 6-2

Problem 3. Lee *second edition* 6-9

Problem 4. Let $F(x_0, x_1, x_2)$ be a homogeneous polynomial of degree k, i.e. it is a linear combination of monomials in the variables x_0, x_1, x_2, each of total degree k. Let $Z(F) \subset \mathbb{RP}^2$ denote the set of points where F vanishes (convince yourself that this vanishing locus is well-defined even though F does not give a well-defined function on $\mathbb{RP}^2$). Prove that $Z(F)$ is an embedded submanifold of $\mathbb{RP}^2$ provided that the partial derivatives $\partial_0 F$, $\partial_1 F$, $\partial_2 F$ do not simultaneously vanish on $Z(F)$.