475 SPRING 2025 PROBLEM SET #8

Problem 1. Find the residue at $z = 0$ of the following functions.

(a) $f(z) = \dfrac{1}{z + z^2}$

(b) $f(z) = z\cos(1/z)$

(c) $f(z) = 1 - \cosh z$

(d) $f(z) = \dfrac{z - \sin z}{z}$

(e) $f(z) = \dfrac{\cot z}{z^4}$

(f) $f(z) = \dfrac{\sinh z}{z^4(1 - z^2)}$

Problem 2. Use the residue formula to evaluate the contour integrals of the following functions around the circle of radius 3 centered at the origin (oriented counterclockwise).

(a) $f(z) = \dfrac{\exp(-z)}{z^2}$

(b) $f(z) = \dfrac{\exp(-z)}{(z-1)^2}$

(c) $f(z) = z^2 \exp(1/z)$

(d) $f(z) = \dfrac{z + 1}{z^2 - 2z}$.

(e) $f(z) = \dfrac{1}{(z-1)(z-2)(2z-1)(z-4)}$

Problem 3. For each of the following functions, write the principal part at the isolated singular point and determine whether it is a removable singularity, pole, or essential singularity.

(a) $f(z) = z\exp(1/z)$

(b) $f(z) = \dfrac{z^2}{1 + z}$

(c) $f(z) = \dfrac{\sin z}{z}$

(d) $f(z) = \dfrac{\cos z}{z}$

(e) $f(z) = \dfrac{1}{(2-z)^3}$.

Problem 4. Show that each of the following functions has a pole, and compute the order of the pole and the corresponding residue.

(a) $f(z) = \dfrac{1 - \cosh z}{z^3}$

(b) $f(z) = \dfrac{1 - \exp(2z)}{z^4}$

(c) $f(z) = \dfrac{\exp(2z)}{(z-1)^2}$

Problem 5. Suppose that f is an entire function, and put $g(z) = \dfrac{f(z)}{z - z_0}$ for some $z_0 \in \mathbb{C}$. Determine the singularity type and residue of $g(z)$ at $z = z_0$.