

475 SPRING 2025 PROBLEM SET #4

Problem 1. Show that $|\exp(2z + i)| + \exp(iz^2) \leq e^{2x} + e^{-2xy}$.

Problem 2. Show that $|\exp(z^2)| \leq \exp(|z|^2)$.

Problem 3.

- (a) Compute $(-1)^{1/\pi}$.
- (b) Compute $i^{1/5}$.
- (c) Compute $i^{(i^i)}$.
- (d) Compute $(1 + i)^i$.

Problem 4.

- (a) Show that the set of values of $\log(i^{1/2})$ is the same as the set of values for $\frac{1}{2} \log(i)$.
- (b) Show that the set of values of $\log(i^2)$ is *not* the same as the set of values for $2 \log(i)$.

Problem 5. Find the principal values of the following:

- (a) i^i
- (b) $(1 - i)^{4i}$.

Problem 6. Show that if we take principal values, then we have $z^{c_1}/z^{c_2} = z^{c_1 - c_2}$.

Problem 7. Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be an entire function, and put $g(z) = \lambda^{f(z)}$ for $\lambda \in \mathbb{C}$. What is $g'(z)$?

Problem 8. Find all solutions to the equation $\sin(z) = \cosh(4)$.

Problem 9. Evaluate the following integrals

- (a) $\int_1^2 \left(\frac{1}{t} - i\right)^2 dt$
- (b) $\int_0^{\pi/6} e^{i2t} dt$
- (c) $\int_0^\infty e^{-zt} dt, \quad \operatorname{Re}(z) > 0.$

Problem 10. Let C denote the unit circle in the complex plane with its counterclockwise orientation. For each integer $k \in \mathbb{Z}$, compute $\int_C f(z) dz$ where $f(z) = z^k$.