434 FALL 2022 PROBLEM SET #3

Problem 1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T(x) = Ax$, where A is a 2×2 orthogonal matrix. Prove that T is either a rotation or a reflection.

Problem 2. Prove that Euclidean isometries (in any dimension) send affine lines (i.e. straight lines not necessarily passing through the origin) to affine lines.

Problem 3. Given any two affine lines in $\mathbb{R}^3$, prove that there is a Euclidean isometry sending one to the other. Prove the same for affine planes.

Problem 4. Consider four points $p_1, p_2, q_1, q_2 \in \mathbb{R}^2$, such that $||p_2 - p_1|| = ||q_2 - q_1||$. How many Euclidean isometries $T : \mathbb{R}^2 \to \mathbb{R}^2$ (if any) are there such that $T(p_1) = q_1$ and $T(p_2) = q_2$?

Problem 5. How many Euclidean isometries $T : \mathbb{R}^n \to \mathbb{R}^n$ are there which have $(0, \ldots, 0)$ as a fixed point (i.e. T maps to origin to itself) and which map the set of unit basis vectors $\{e_1 := (1, 0, \ldots, 0), \ldots, e_n := (0, \ldots, 0, 1)\}$ to itself?