434 FALL 2022 PROBLEM SET #2

Problem 1. Hitchman 3.1.2

Problem 2. Hitchman 3.1.4

Problem 3. Hitchman 3.1.6

Problem 4. Hitchman 3.1.10

Problem 5. Let X be the plane $\mathbb{R}^2$, and let $\mu_1, \mu_2, \mu_3, \mu_4 : X \times X \to \mathbb{R}$ be given by

$$\mu_1((x,y),(x',y')) := \sqrt{(x-x')^2 + (y-y')^2}$$
$$\mu_2((x,y),(x',y')) := |x-x'| + |y-y'|$$
$$\mu_3((x,y),(x',y')) := \max\{|x-x'|, |y-y'|\}$$
$$\mu_4((x,y),(x',y')) := (\sqrt{|x-x'|} + \sqrt{|y-y'|})^2.$$

For $i = 1, 2, 3, 4$, prove or disprove that (X, μ_i) is a metric space.

Problem 6. If (X, μ) is a metric space, the *ball of radius $r > 0$ centered at the point $p \in X$* is the set

$$B_\mu(p, r) := \{q \in X \mid \mu(p, q) < r\},$$

i.e. those points whose distance from p is less than r. For each of the examples in the previous problem, draw the ball of radius 1 centered at $(0, 0) \in \mathbb{R}^2$.