425A FALL 2020 PROBLEM SET #8

Problem 1. Pugh (2nd edition) chapter 2 problem 37.

Problem 2. Pugh (2nd edition) chapter 2 problem 47(a).

Problem 3. Let (M, d) be a metric space, and $S \subset M$ a connected subset. Is the interior of S connected? Prove or give a counterexample.

Problem 4. Read the section "Clustering and Condensing" in Pugh.

(a) Find a bounded set of real numbers with exactly three cluster points.
(b) Find a compact set of real numbers whose set of cluster points is countably infinite.

Problem 5. Read the section "Continuity of Arithmetic in $\mathbb{R}$" in Pugh. Prove that the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x, y) = (\sin(xy^2), 3x^3y + xy^2)$ is continuous.

Problem 6. Does a continuous function between two metric spaces send closed subsets to closed subsets? Prove or give a counterexample.

Problem 7. Give an example of an open cover of $(0, 1)$ which has no finite subcover. Conclude that $(0, 1)$ is not compact.

Problem 8. Let $S \subset \mathbb{R}^2$ be the "closed topologist's sine curve", defined by

$$S := \{(x, \sin(1/x)) \in \mathbb{R}^2 \ : \ x \in (0, 1]\} \cup \{(0, y) \ : \ -1 \leq y \leq 1\}.$$

Prove that S is connected but not path connected.