425A FALL 2020 PROBLEM SET #7

Problem 1.

(a) Let (M, d) be a metric space, and let $S \subset M$ be a subset. Recall that the closure of S in M, denoted by $\overline{S}$, is by definition the set of all limits of S in M. Prove that $\overline{S}$ is closed in M, that it contains S, and that it is the smallest closed subset of M which contains S.

(b) Let (M, d) be a metric space, and let $S \subset M$ be a subset. A point $p \in M$ is called an *interior point* if there is some $r > 0$ such that the $B_r(p) \subset S$.[1] The *interior of* S, denoted by $\operatorname{int}(S)$, is definition of the set of all interior points of S. Prove that S is an open subset of M if and only if $S = \operatorname{int}(S)$.

Problem 2. Pugh (2nd edition) chapter 2 problem 28.

Problem 3. Pugh (2nd edition) chapter 2 problem 30.

Problem 4. Pugh (2nd edition) chapter 2 problem 34.

Problem 5. Pugh (2nd edition) chapter 2 problem 38.

Problem 6. Pugh (2nd edition) chapter 2 problem 39.

Problem 7. Pugh (2nd edition) chapter 2 problem 43.

[1] Recall that $B_r(p)$ is the open ball centered at p of radius r, i.e. $B_r(p) = \{x \in M \ : \ d(x, p) < r\}$.