425A FALL 2020 PROBLEM SET #6

Problem 1. Recall that we gave our definition of continuity for a function $f : M \to N$ between two metric spaces in terms of ϵ and δ. We also defined "sequential continuity" in class: f sends convergent sequences in M to convergent sequences in N, with limits being sent to limits. You proved in the last problem set that the part about limits being sent to limits is actually redundant. We proved in class that continuity is equivalent to sequential continuity[1]. We also defined "topological continuity" in class: the preimage of any open subset of N is an open subset of M.

a Write out a careful proof that continuity is equivalent to topological continuity.
b Suppose that we were to swap "open" with "closed" in the definition of topological continuity. Prove that the resulting definition would be equivalent.

Problem 2. Let (M, d_M) and (N, d_N) be metric spaces.

a Suppose that d_M is the discrete metric.[2] Describe all continuous functions from M to N. You should rigorously justify your answer.
b Now suppose instead that d_N is the discrete metric. Assume that (M, d_M) is *connected* (see Pugh page 86 for the definition). Describe all continuous functions from M to N. You should rigorously justify your answer.

Problem 3. Pugh (2nd edition) chapter 2 problem 18.

Problem 4. Pugh (2nd edition) chapter 2 problem 19.

Problem 5. Pugh (2nd edition) chapter 2 problem 23.

Problem 6. Pugh (2nd edition) chapter 2 problem 25.

[1]Pugh actually reverses the definitions of continuity and sequential continuity in the 2nd edition of his book for some reason. In the end it doesn't make much difference, since we've proven that these two concepts are equivalent, even though it's not obvious from the get-go.

[2]Recall that discrete metric is the one for which the distance between any two distinct points is 1.