425A FALL 2020 PROBLEM SET #4

Problem 1. (1) Let A and B be sets, and suppose that we have maps $f : A \to B$ and $g : B \to A$ such that $g \circ f$ is injective. Is f necessarily injective? What about g?

(2) Let A and B be sets, and suppose that we have maps $f : A \to B$ and $g : B \to A$ such that $g \circ f$ is surjective. If f necessarily surjective? What about g?

(3) Let A and B be sets, and suppose that we have maps $f : A \to B$ and $g : B \to A$ such that f is injective and g is surjective. Is $g \circ f$ necessarily injective? Is it necessarily surjective?

(4) Let A and B be sets, and suppose that we have maps $f : A \to B$ and $g : B \to A$ such that f is injective and g is injective. Is $g \circ f$ necessarily injective?

Problem 2. Let A be any infinite set and B be any countable set. Prove that we have $A \sim A \cup B$.

Problem 3. Pugh (2nd edition) chapter 1 problem 38 (a). Extra credit: 38 (b).

Problem 4. Pugh (2nd edition) chapter 1 problem 39 (a) and (d).

Problem 5. Pugh (2nd edition) chapter 1 problem 40.