

**425A FALL 2020 PROBLEM SET #10**

**Problem 1.** Consider a function  $f : \mathbb{R} \rightarrow \mathbb{R}$  which is differentiable and satisfies  $\lim_{x \rightarrow \infty} f'(x) = 0$ . Prove that we have  $\lim_{x \rightarrow \infty} (f(x+1) - f(x)) = 0$ .

**Problem 2.** Let  $q_1, q_2, q_3, \dots$  be some enumeration of the set of rational numbers in  $(0, 1)$ . Define a function  $f : (0, 1) \rightarrow \mathbb{R}$  by

$$f(x) = \sum_{q_n < x} 2^{-n}.$$

In other words,  $f(x)$  is the sum of  $1/2^n$  over all  $n$  such that  $q_n$  is less than  $x$ . Prove that  $f$  is continuous at every irrational number and discontinuous at every rational number.

**Problem 3.** Pugh (2nd edition) chapter 3 problem 1.

**Problem 4.** Pugh (2nd edition) chapter 3 problem 3.

**Problem 5.** Pugh (2nd edition) chapter 3 problem 4.

**Problem 6.** Pugh (2nd edition) chapter 3 problem 8(a).

**Problem 7.** Pugh (2nd edition) chapter 3 problem 9.

**Problem 8** (Extra credit). Pugh (2nd edition) chapter 1 problem 31.